Pensieve header: Some experiments with capping the buckle in parts.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2012-05\\beta5.1"];
<< betaCalculus.m
Clear[ħ]; Unprotect[C];
$PerturbativeDegree = 8;
βSimplify[expr_] := Replace[
   Series[Normal[expr], {ħ, 0, $PerturbativeDegree}],
   sd_SeriesData :> MapAt[Expand, sd, 3]
  ];
βCollect[B[ω_, μ_]] := B[βSimplify[ω], βSimplify[μ]];
{V0, C0, sol} = Get[Switch[$PerturbativeDegree,
    4, "SolutionToDegree4-120523.m",
    6, "SolutionToDegree6-120523.m",
    8, "SolutionToDegree8-120524.m"
   ]];
C = C0 /. κ1 → 0;
ν = B[Series[Sinh[c1 ħ / 2] / (c1 ħ / 2), {ħ, 0, $PerturbativeDegree}], 0];
Φ0 = (Inverse[V0] // dP[12, 3]) ** Inverse[V0] ** (V0 // dP[2, 3]) ** (V0 // dP[1, 23]);
V = Inverse[C // dP[12]] ** V0 ** (C ** (C // dP[2]));
Φ = (Inverse[V] // dP[12, 3]) ** Inverse[V] ** (V // dP[2, 3]) ** (V // dP[1, 23]);
CC = C ** C;
Clear[C];
```

```
Φ == Φ0
```

```
True
```

```
DeleteCases[{
  "Test" → xxx == yyy,
  "R4" → R[2, 3] ** R[1, 3] ** V == V ** (R[1, 3] // dΔ[1, 1, 2]),
  "TwistEq" → V ** Θ[1, 2] == R[1, 2] ** (V // dP[2, 1]),
  "Unitarity" → V ** (CC // dP[12]) ** (V // dA[1] // dA[2]) == CC ** (CC // dP[2]),
  "VerticalFlipForV" →
   V ** (CC // dP[12]) ** (V // dS[1] // dS[2]) == R[1, 2] ** CC ** (CC // dP[2]),
  "CapEquation" → ((V ** (CC // dP[12])) // dcap[1] // dcap[2]) == CC ** (CC // dP[2]),
  "VSidesDelete" → (V // dη[1]) == B[1, 0] && (V // dη[2]) == B[1, 0],
  "CapsAndCups" → CC == (CC // dS[1]),
  "Pentagon" → Φ ** (Φ // dP[1, 23, 4]) ** (Φ // dP[2, 3, 4]) ==
    (Φ // dP[12, 3, 4]) ** (Φ // dP[1, 2, 34]),
  "PositiveHexagon" → (Θ[1, 2, +1] // dP[12, 3]) ==
    (Φ ** Θ[2, 3, +1] ** Inverse[Φ // dP[1, 3, 2]] ** Θ[1, 3, +1] ** (Φ // dP[3, 1, 2])),
  "NegativeHexagon" → (Θ[1, 2, -1] // dP[12, 3]) ==
    (Φ ** Θ[2, 3, -1] ** Inverse[Φ // dP[1, 3, 2]] ** Θ[1, 3, -1] ** (Φ // dP[3, 1, 2])),
  "HorizontalFlipForΦ" → Φ ** (Φ // dP[3, 2, 1]) == B[1, 0],
  "VerticalFlipForΦ" → Φ ** (Φ // dS[1] // dS[2] // dS[3]) == B[1, 0],
  "OverhandEquation" →
   (Φ // dΔ[1, 0, 1] // dS[2] // dS[3] // dm[0, 3, 0] // dm[1, 2, 1]) == B[1, 0],
  "ValueOfν" → (Φ // dS[2] // dm[3, 2, 2] // dm[2, 1, 1]) == ν,
  "ValueOfCC" → CC ** CC == Inverse[ν],
  "VTopDelete" → (V // dS[1] // dm[2, 1, 1]) == R[1, 1, -1 / 2],
  "EKTopCapLeftPuncture" →
   (V // tη[1] // dm[2, 3, 2] // dS[2] // hm[1, 2, 1]) == B[1, 0],
  "EKRightCupLeftPuncture" →
   (V // dm[3, 2, 2] // hη[2] // tη[1] // dm[1, 2, 1]) == B[1, 0],
  "EKRightCupTopPuncture" →
   (V // dm[3, 2, 2] // hη[2] // dS[1] // dm[2, 1, 1]) == B[1, 0],
  "EKTopCapRightPuncture" →
   (V // tη[2] // dm[1, 3, 1] // dS[1] // dm[2, 1, 1]) == R[1, 1, -1 / 2],
  "EKLeftCupRightPuncture" →
   (V // dm[3, 1, 1] // hη[1] // tη[2] // dm[2, 1, 1]) == R[1, 1, 1 / 2],
  "EKLeftCupTopPuncture" → (V // dm[3, 1, 1] // hη[1] // dS[1] // dm[2, 1, 1]) ==
    R[1, 1, -1 / 2],
  "BuckleEquation" → (
    buckle = (Inverse[Φ] // dP[13, 2, 4]) **
      (Φ // dP[1, 3, 2]) ** Θ[3, 2] ** Inverse[Φ] ** (Φ // dP[12, 3, 4]);
    LuckyV = buckle // tη[1] // hη[2] // dm[1, 2, 1] // tη[3] // hη[4] // dm[3, 4, 2];
    V == LuckyV
   )
 }, _ → True]
```

```
{Test → xxx == yyy}
```

```
{V // dcap[1] // tη[2],
  V // dcap[2] // tη[1]} // ColumnForm
```

$$\begin{pmatrix} 1 & h[2] \\ t[1] & \frac{1}{2}+\frac{c_1\hbar}{8}+\frac{1}{48}c_1^2\hbar^2+\frac{1}{384}c_1^3\hbar^3+\frac{c_1^4\hbar^4}{3840}+\frac{c_1^5\hbar^5}{46\,080}+\frac{c_1^6\hbar^6}{645\,120}+\frac{c_1^7\hbar^7}{10\,321\,920}+\frac{c_1^8\hbar^8}{185\,794\,560}+O[\hbar]^9 \end{pmatrix}$$

```
(1)
```

CC ⋆⋆ (CC // dP[2]) ⋆⋆ Inverse[CC // dP[12]]

$$\begin{pmatrix} 1+\frac{1}{24}c_1c_2\hbar^2+\left(-\frac{c_1^3c_2}{1440}-\frac{c_1^2c_2^2}{5760}-\frac{c_1c_2^3}{1440}\right)\hbar^4+\left(\frac{c_1^5c_2}{60480}+\frac{c_1^4c_2^2}{80640}+\frac{23c_1^3c_2^3}{967680}+\frac{c_1^2c_2^4}{80640}+\frac{c_1c_2^5}{60480}\right)\hbar^6+\left(-\frac{c_1^7c_2}{2419200}-\frac{c_1^6c_2^2}{1935} \\ t[1] \\ t[2] \end{pmatrix}$$

((Inverse[Φ] // dP[13, 2, 4]) ⋆⋆ (Φ // dP[1, 3, 2]) ⋆⋆ Θ[3, 2] ⋆⋆ Inverse[Φ] ⋆⋆ (Φ // dP[12, 3, 4])) // tη[1] // hη[2] // dm[1, 2, 1] // tη[3] // hη[4] // dm[3, 4, 2] // dcap[1] // dcap[2]

$$\begin{pmatrix} 1+\frac{1}{24}c_1c_2\hbar^2+\left(-\frac{c_1^3c_2}{1440}-\frac{c_1^2c_2^2}{5760}-\frac{c_1c_2^3}{1440}\right)\hbar^4+\left(\frac{c_1^5c_2}{60480}+\frac{c_1^4c_2^2}{80640}+\frac{23c_1^3c_2^3}{967680}+\frac{c_1^2c_2^4}{80640}+\frac{c_1c_2^5}{60480}\right)\hbar^6+\left(-\frac{c_1^7c_2}{2419200}-\frac{c_1^6c_2^2}{1935} \\ t[1] \\ t[2] \end{pmatrix}$$

((Inverse[Φ] // dP[13, 2, 4])) // tη[1] // hη[2] // dm[1, 2, 1] // tη[3] // hη[4] // dm[3, 4, 2]

$$\begin{pmatrix} 1 & & h[1] \\ t[1] & \frac{c_2\hbar}{24}+\left(-\frac{7c_1^2c_2}{5760}-\frac{7c_1c_2^2}{5760}-\frac{c_2^3}{1440}\right)\hbar^3+\left(\frac{31c_1^4c_2}{967680}+\frac{31c_1^3c_2^2}{483840}+\frac{83c_1^2c_2^3}{967680}+\frac{13c_1c_2^4}{241920}+\frac{c_2^5}{60480}\right)\hbar^5+\left(-\frac{127c_1^6c_2}{154828800}-\frac{1}{51}\right. \\ t[2] & -\frac{c_1\hbar}{24}+\left(\frac{7c_1^3}{5760}+\frac{7c_1^2c_2}{5760}+\frac{c_1c_2^2}{1440}\right)\hbar^3+\left(-\frac{31c_1^5}{967680}-\frac{31c_1^4c_2}{483840}-\frac{83c_1^3c_2^2}{967680}-\frac{13c_1^2c_2^3}{241920}-\frac{c_1c_2^4}{60480}\right)\hbar^5+\left(\frac{127c_1^7}{154828800}+\frac{12}{51}\right. \end{pmatrix}$$

((Inverse[Φ] // dP[13, 2, 4])) // tη[1] // hη[2] // dm[1, 2, 1] // tη[3] // hη[4] // dm[3, 4, 2] // dcap[1] // dcap[2]

(1)

((Φ // dP[1, 3, 2])) // tη[1] // hη[2] // dm[1, 2, 1] // tη[3] // hη[4] // dm[3, 4, 2]

(1)

(Θ[3, 2]) // tη[1] // hη[2] // dm[1, 2, 1] // tη[3] // hη[4] // dm[3, 4, 2]

$$\begin{pmatrix} 1 & & h[2] \\ t[1] & \frac{1}{2}+\frac{c_1\hbar}{8}+\frac{1}{48}c_1^2\hbar^2+\frac{1}{384}c_1^3\hbar^3+\frac{c_1^4\hbar^4}{3840}+\frac{c_1^5\hbar^5}{46080}+\frac{c_1^6\hbar^6}{645120}+\frac{c_1^7\hbar^7}{10321920}+\frac{c_1^8\hbar^8}{185794560}+O[\hbar]^9 \end{pmatrix}$$

(Θ[3, 2]) // tη[1] // hη[2] // dm[1, 2, 1] // tη[3] // hη[4] // dm[3, 4, 2] // dcap[1] // dcap[2]

(1)

(Inverse[Φ]) // tη[1] // hη[2] // dm[1, 2, 1] // tη[3] // hη[4] // dm[3, 4, 2]

(1)

((Φ // dP[12, 3, 4])) // tη[1] // hη[2] // dm[1, 2, 1] // tη[3] // hη[4] // dm[3, 4, 2]

$$\begin{pmatrix} 1 & & h[2] \\ t[1] & \frac{c_2\hbar}{24}+\left(-\frac{c_1^2c_2}{1440}-\frac{c_1c_2^2}{5760}-\frac{c_2^3}{1440}\right)\hbar^3+\left(\frac{c_1^4c_2}{60480}+\frac{c_1^3c_2^2}{80640}+\frac{23c_1^2c_2^3}{967680}+\frac{c_1c_2^4}{80640}+\frac{c_2^5}{60480}\right)\hbar^5+\left(-\frac{c_1^6c_2}{2419200}-\frac{c_1^5c_2^2}{1935360}-\right. \\ t[2] & -\frac{c_1\hbar}{24}+\left(\frac{c_1^3}{1440}+\frac{c_1^2c_2}{5760}+\frac{c_1c_2^2}{1440}\right)\hbar^3+\left(-\frac{c_1^5}{60480}-\frac{c_1^4c_2}{80640}-\frac{23c_1^3c_2^2}{967680}-\frac{c_1^2c_2^3}{80640}-\frac{c_1c_2^4}{60480}\right)\hbar^5+\left(\frac{c_1^7}{2419200}+\frac{c_1^6c_2}{1935360}+\right. \end{pmatrix}$$

((Φ // dP[12, 3, 4])) // tη[1] // hη[2] // dm[1, 2, 1] // tη[3] // hη[4] // dm[3, 4, 2] // dcap[1] // dcap[2]

$$\begin{pmatrix} 1+\frac{1}{24}c_1c_2\hbar^2+\left(-\frac{c_1^3c_2}{1440}-\frac{c_1^2c_2^2}{5760}-\frac{c_1c_2^3}{1440}\right)\hbar^4+\left(\frac{c_1^5c_2}{60480}+\frac{c_1^4c_2^2}{80640}+\frac{23c_1^3c_2^3}{967680}+\frac{c_1^2c_2^4}{80640}+\frac{c_1c_2^5}{60480}\right)\hbar^6+\left(-\frac{c_1^7c_2}{2419200}-\frac{c_1^6c_2^2}{1935} \\ t[1] \\ t[2] \end{pmatrix}$$

```
CC ** (CC // dP[2]) ** Inverse[CC // dP[12]]
```

$$\left(1+\frac{1}{24}c_1c_2\hbar^2+\left(-\frac{c_1^3c_2}{1440}-\frac{c_1^2c_2^2}{5760}-\frac{c_1c_2^3}{1440}\right)\hbar^4+\left(\frac{c_1^5c_2}{60\,480}+\frac{c_1^4c_2^2}{80\,640}+\frac{23c_1^3c_2^3}{967\,680}+\frac{c_1^2c_2^4}{80\,640}+\frac{c_1c_2^5}{60\,480}\right)\hbar^6+\left(-\frac{c_1^7c_2}{2\,419\,200}-\frac{c_1^6c_2^2}{1\,935\,3}\right.\right.$$
$$\left.\left.\begin{matrix}t[1]\\t[2]\end{matrix}\right.$$

```
Φ // tη[2] // dm[2, 3, 2] // dcap[2]
```

$$\left(1+\frac{1}{24}c_1c_2\hbar^2+\left(-\frac{c_1^3c_2}{1440}-\frac{c_1^2c_2^2}{5760}-\frac{c_1c_2^3}{1440}\right)\hbar^4+\left(\frac{c_1^5c_2}{60\,480}+\frac{c_1^4c_2^2}{80\,640}+\frac{23c_1^3c_2^3}{967\,680}+\frac{c_1^2c_2^4}{80\,640}+\frac{c_1c_2^5}{60\,480}\right)\hbar^6+\left(-\frac{c_1^7c_2}{2\,419\,200}-\frac{c_1^6c_2^2}{1\,935\,3}\right.\right.$$
$$\left.\left.\begin{matrix}t[1]\\t[2]\end{matrix}\right.$$

```
Φ // tη[2] // dm[2, 3, 2]
```

$$\begin{pmatrix}1 & & & h[2]\\ t[1] & \frac{c_2\hbar}{24}+\left(-\frac{c_1^2c_2}{1440}-\frac{c_1c_2^2}{5760}-\frac{c_2^3}{1440}\right)\hbar^3+\left(\frac{c_1^4c_2}{60\,480}+\frac{c_1^3c_2^2}{80\,640}+\frac{23c_1^2c_2^3}{967\,680}+\frac{c_1c_2^4}{80\,640}+\frac{c_2^5}{60\,480}\right)\hbar^5+\left(-\frac{c_1^6c_2}{2\,419\,200}-\frac{c_1^5c_2^2}{1\,935\,360}-\right.\\ t[2] & -\frac{c_1\hbar}{24}+\left(\frac{c_1^3}{1440}+\frac{c_1^2c_2}{5760}+\frac{c_1c_2^2}{1440}\right)\hbar^3+\left(-\frac{c_1^5}{60\,480}-\frac{c_1^4c_2}{80\,640}-\frac{23c_1^3c_2^2}{967\,680}-\frac{c_1^2c_2^3}{80\,640}-\frac{c_1c_2^4}{60\,480}\right)\hbar^5+\left(\frac{c_1^7}{2\,419\,200}+\frac{c_1^6c_2}{1\,935\,360}+\right.\end{pmatrix}$$